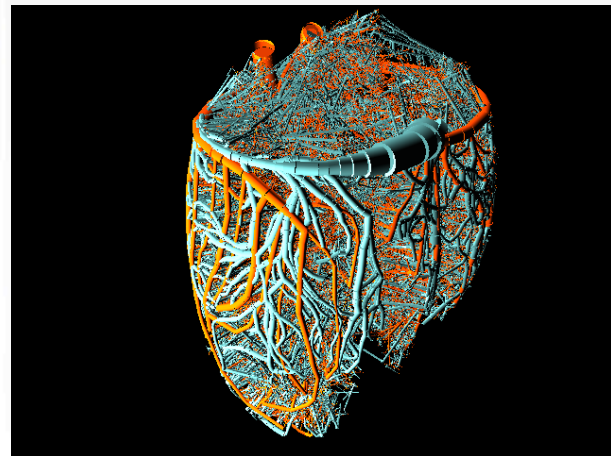
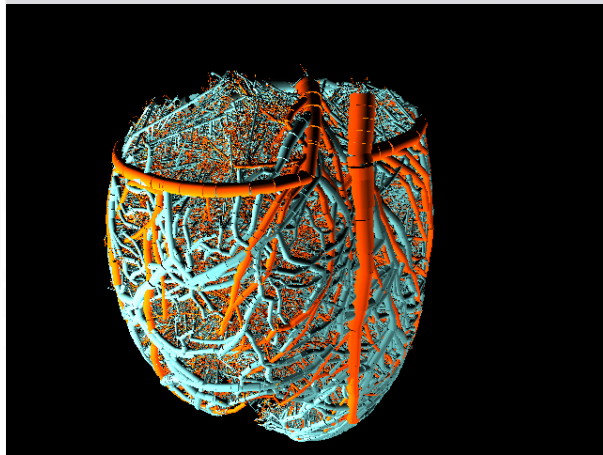


Mathematical Modeling of Coronary Arterial Tree

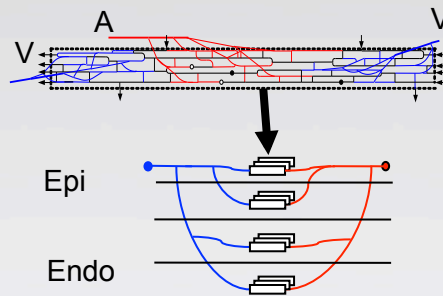
G. S. Kassab

California Medical Innovations Institute
San Diego, CA, USA.

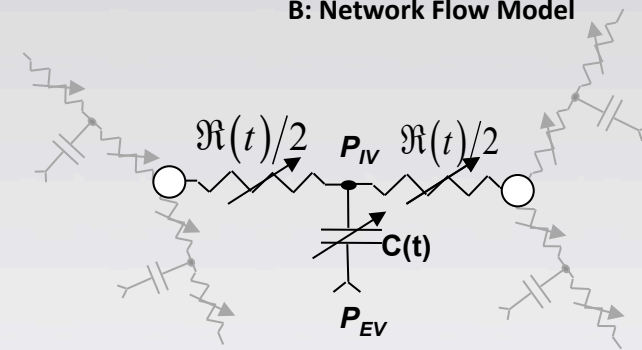
Entire Coronary Vasculature



A: Network Reconstruction



B: Network Flow Model

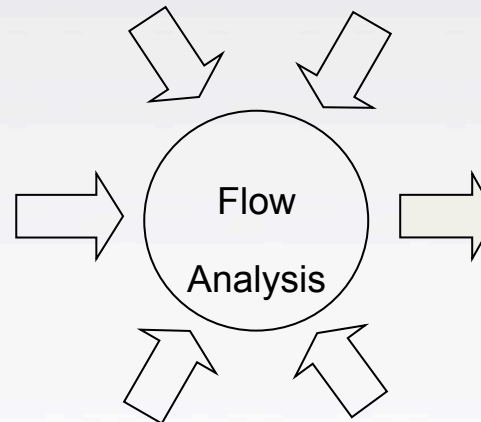


E: Inputs

- * HR
- * Contractility
- * LVP(t)
- * $P_{perf}(t)$
- * $P_v(t)$

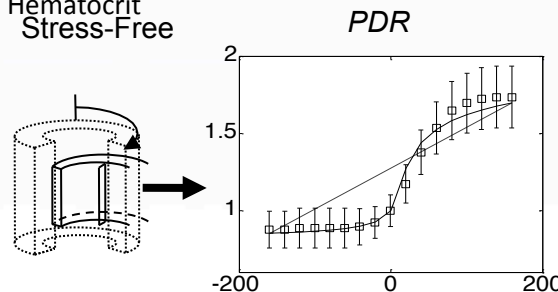
F: Outputs

- * $\Delta P(t)$
- * Regional Perfusion

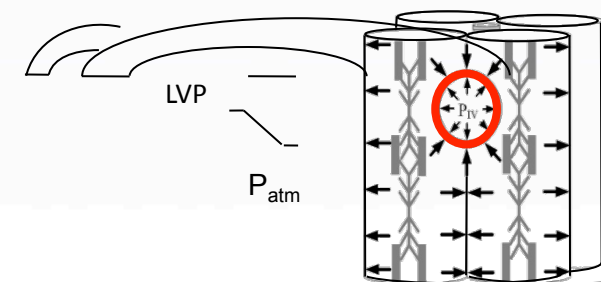


C: Vessel-in-Myocardium Model

- * Hematocrit Stress-Free



D: Muscle Vessel Interaction



Mathematical Rules of Bifurcation Diameters

Murray's $D_m^3 = D_l^3 + D_s^3$ (WSS \sim Constant)

HK $D_m^{\frac{7}{3}} = D_l^{\frac{7}{3}} + D_s^{\frac{7}{3}}$

Area-Preservation $D_m^2 = D_l^2 + D_s^2$ (Velocity \sim Constant)

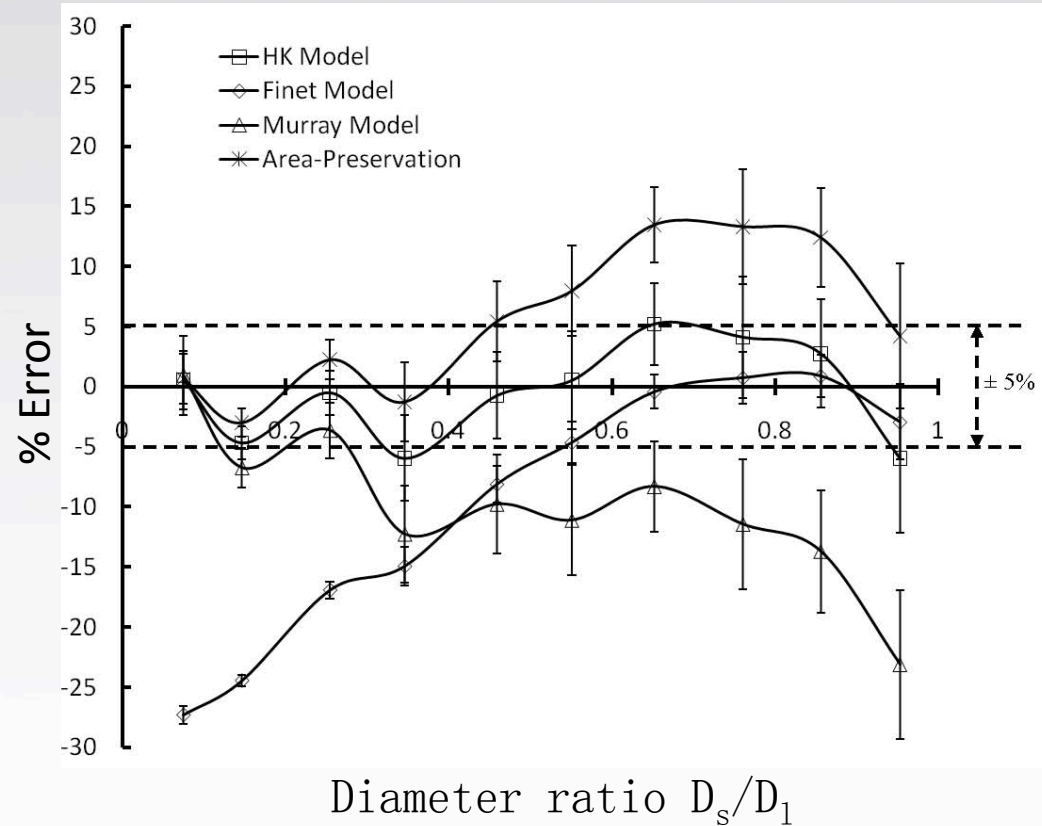
Finet $D_m = 0.678(D_l + D_s)$

where D_m , D_l and D_s are diameter of mother, larger and smaller daughter vessels, respectively.

Murray and HK (Huo-Kassab) models are based on minimum energy hypothesis. Area-preservation and Finet Models are based on fractal models.

Diameter Models vs. Measurements

Mother vessels of various sizes	% Error			
	HK	Finet	Murray	Area- Preservation
For all	0.57	0.39	-16.62	10.62
$4.5 \leq D_m$	-1.33	-0.36	-18.73	8.86
$4 \leq D_m < 4.5$	-2.03	-0.58	-19.57	8.25
$3.5 \leq D_m < 4$	1.02	0.48	-16.02	10.99
$3 \leq D_m < 3.5$	-5.6	-2.59	-22.87	4.60
$2.5 \leq D_m < 3$	8.53	3.65	-7.96	18.04
$D_m \leq 2.5$	8.77	4.08	-8.14	18.44



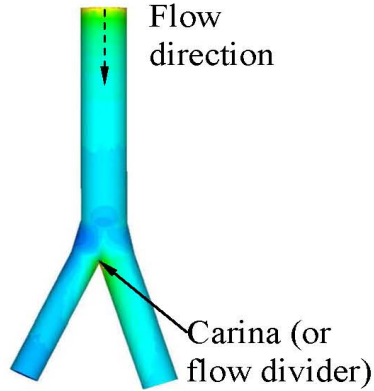
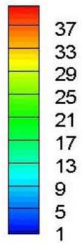
Left Table: Values obtained from quantitative coronary bifurcation angiography - Y bifurcations

Right Figure: Values obtained from morphometric casts - all bifurcations (Y and T bifurcations)

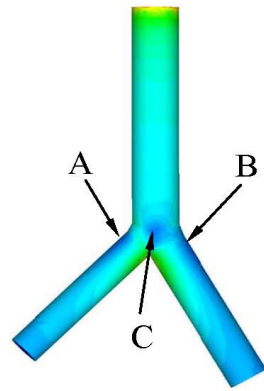
Huo, Y Finet G, Lefevre T, Louvard Y, Moussa I, and G.S. Kassab. *EuroIntervention* 20;7(11):1310-6, 2012.

WSS and OSI at Y-type Bifurcations

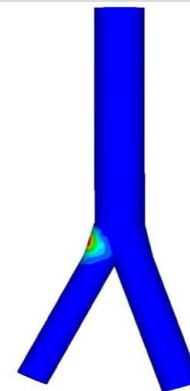
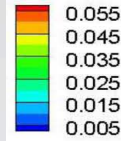
Dynes/cm²



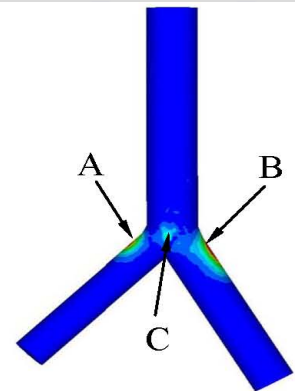
WSS-HK



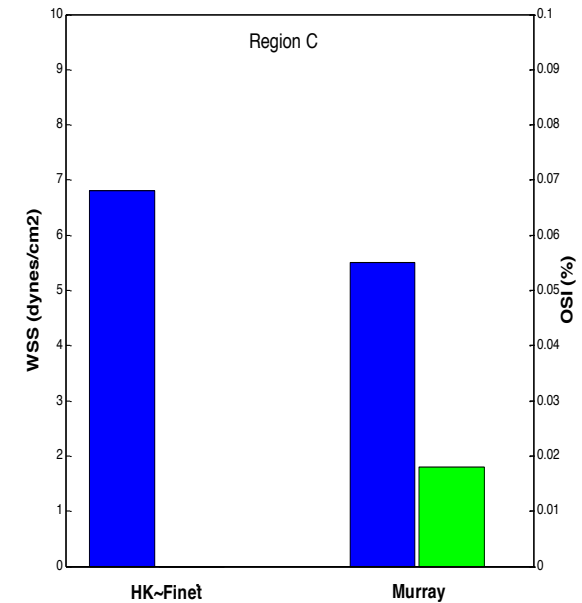
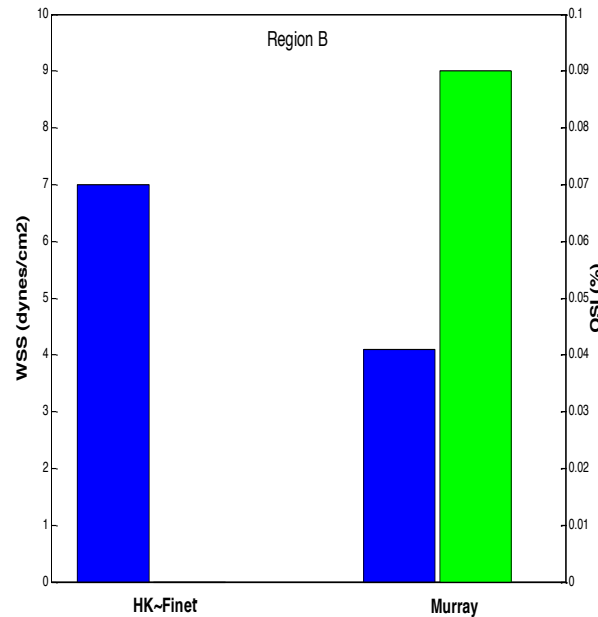
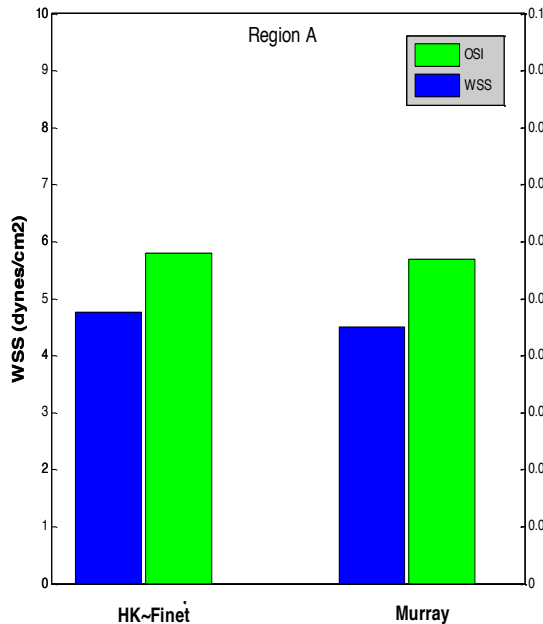
WSS-Murray



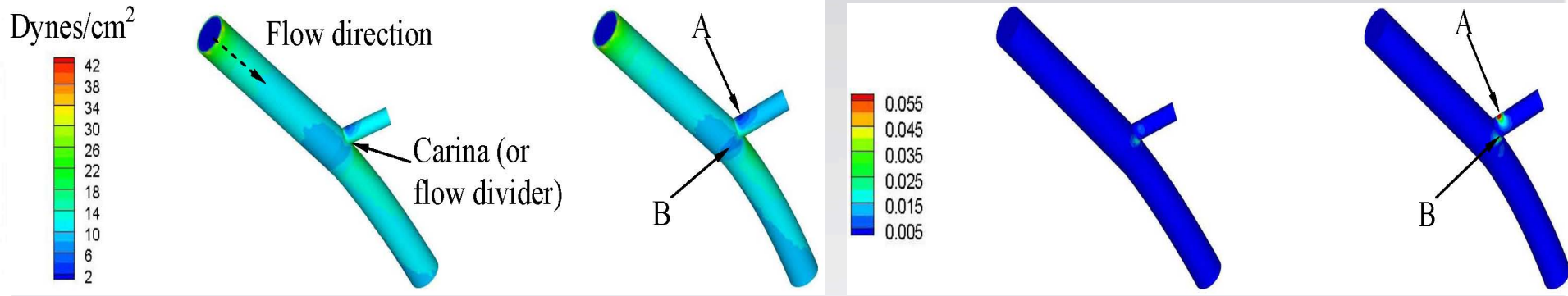
OSI-HK



OSI-Murray



WSS and OSI at T-type Bifurcations

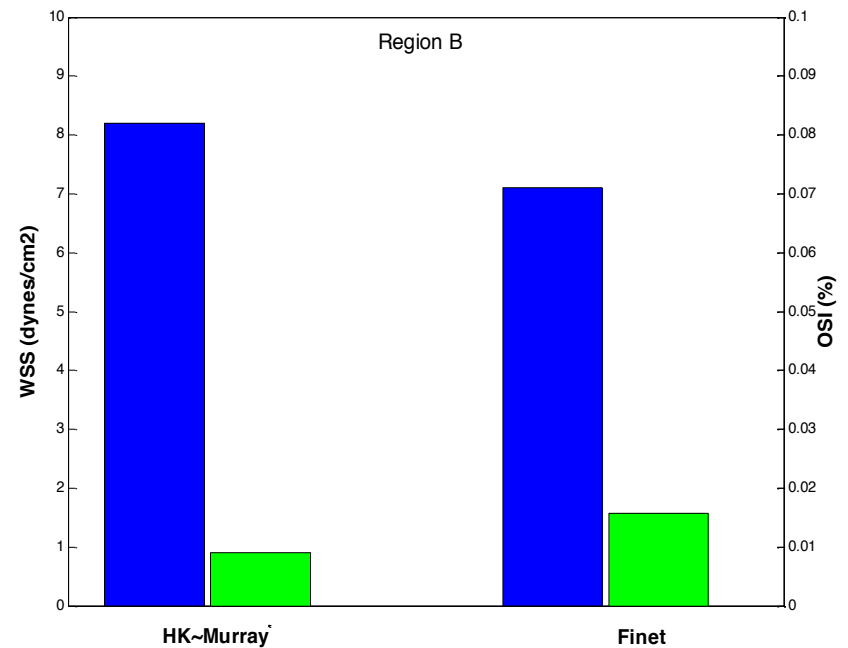
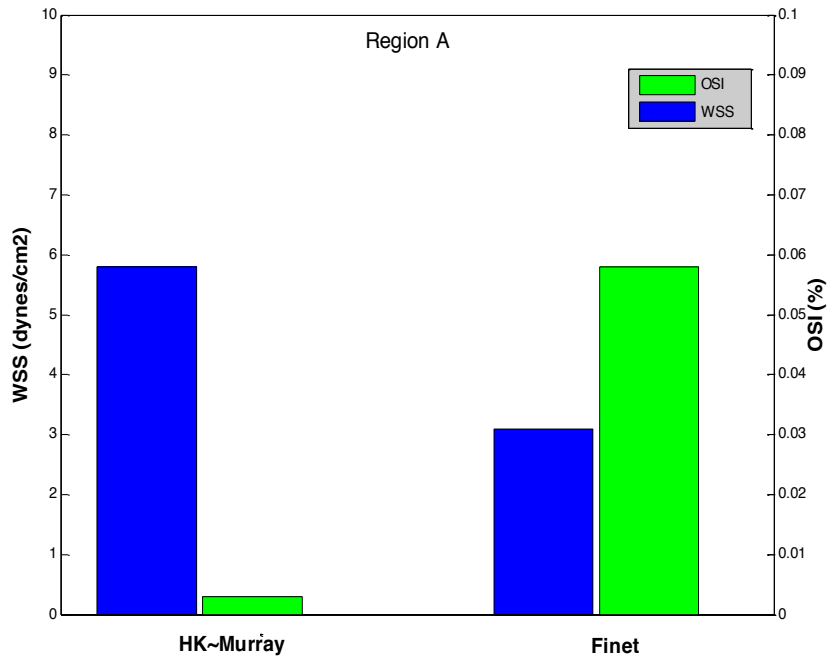


WSS-HK

WSS-Finet

OSI-HK

OSI-Finet



Conclusions

- HK model is in agreement with measurements of all bifurcations types (e.g., Y and T) and provides high WSS and low OSI.
- Murray and area-preservation models are not in agreement with experimental measurements. Murray's law provides lower WSS and higher OSI which challenges the previous assumed notion that Murray's bifurcation angle is ideal for optimal hemodynamics.
- Finet is in agreement for Y bifurcations but not for T-bifurcations where it leads to lower WSS and higher OSI.
- We have implemented an easy to use rule to determine the 3rd diameter of bifurcation/trifurcation (if two/three of the segment diameters are known) that can be downloaded to an iPhone, BlackBerry: <http://ett.calmi2.org/cnc/refdiacalculation>

Example (bifurcation): D_m : D_1 : D_s :

GS Kassab and G Finet. *EuroIntervention* 19;11 Suppl V:V13-V17, 2015.